Code No. **041**

Roll No. | | | | | | | |

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Paper (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. Write the smallest equivalence relation R on Set A = {1, 2, 3}.

Or

Give an example to show that the relation R in the set of natural numbers, defined by $R = \{(x, y), x, y \in \mathbf{N}, x \le y^2\}$ is not transitive.

2. Write the number of all one-one functions from the set A = $\{a, b, c\}$ to itself.
3. Let A = {1, 2, 3, 4}. Let R be the equivalence relation on A × A defined by $(a, b)\ R(c, d)$ if $a + d = b + c$. Find the equivalence class [(1, 3)].

Or

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

4. If $A = \begin{bmatrix} 4 & 6 \\ 7 & 5 \end{bmatrix}$, then what is A. (*adj*.A)?
5. For what value of k, the matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is skew symmetric?

Or

If $\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = \frac{1}{2}$, where α, β are acute angles, then write the value of α + β.

6. If A is a square matrix of order 3 such that $|adj\ A| = 225$, find $|A'|$.
7. If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$, write the value of k.

Or

Evaluate : $\int \cot x\,(\operatorname{cosec} x - 1)\, e^x\, dx$

8. Write the value of : $\left(\dfrac{dy}{dx}\right)^3 dx$

9. If m and n are the order and degree, respectively of the differential equation $y\left(\dfrac{dy}{dx}\right)^3 + x^3\left(\dfrac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of $m + n$.

Or

Find the integral factor of differential equation : $\sec x \dfrac{dy}{dx} - y = \sin x$.

10. If $|\vec{a}| = a$, then find the value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$.

11. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then write the value of $|\vec{a} + \vec{b}|$.

12. If $\vec{a}$ and $\vec{b}$ are two vector of magnitude 3 and $\dfrac{2}{3}$ respectively such that $\vec{a} \times \vec{b}$ is a unit vector, write the angle between $\vec{a}$ and $\vec{b}$.

13. Find the distance of the point (a, b, c) from x-axis.

14. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.

15. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are $\dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}$ and $\dfrac{2}{3}$. What is the probability that at most one of them will solve the problem?

16. If $P(A) = \dfrac{2}{5}$, $P(B) = \dfrac{1}{3}$, $P(A \cap B) = \dfrac{1}{5}$, then find $P(\overline{A} / \overline{B})$.

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17–21) and (22 – 26). Each question carries 1 mark.

17. Case Study—Factories that used to make perfumes,T-shirts and cars now making supplies to fight against the Corona virus. Manufactures, fashion designers, and 3D printing companies are now making face masks, PPE kits, gloves, ventilators and hand sanitizers.

A factory has three machines I, II and III which produce 30%, 50%, 20% respectively of the total items of the same variety. Out of these 2%, 5% and 3% respectively are found to be defective.

Answer the following questions :

(*i*) Probability of production by the three machines is:

(*a*) $\dfrac{3}{10}, \dfrac{5}{10}, \dfrac{2}{10}$ (*b*) $\dfrac{3}{100}, \dfrac{5}{100}, \dfrac{2}{100}$ (*c*) $\dfrac{30}{10}, \dfrac{50}{10}, \dfrac{20}{10}$ (*d*) $\dfrac{0.3}{100}, \dfrac{0.5}{100}, \dfrac{0.2}{100}$

(*ii*) Probability of production of three machines if an item is picked and found to be defective is:

(*a*) $\dfrac{2}{10}, \dfrac{5}{10}, \dfrac{3}{10}$ (*b*) $\dfrac{0.2}{100}, \dfrac{0.5}{100}, \dfrac{0.3}{100}$ (*c*) $\dfrac{2}{100}, \dfrac{5}{100}, \dfrac{3}{100}$ (*d*) $\dfrac{20}{10}, \dfrac{50}{10}, \dfrac{30}{10}$

(*iii*) Probability that the defective item is produced by machine III is:

(*a*) $\dfrac{3}{37}$ (*b*) $\dfrac{6}{37}$ (*c*) $\dfrac{2}{37}$ (*d*) $\dfrac{10}{37}$

(*iv*) Which of the following would satisfy the condition that Machine I and II are independent events?

(*a*) $P(I \cap II) = P(I) . P(II)$ (*b*) $P(I \cup II) = P(I) . P(II)$
(*c*) $P(I' \cap II) = P(I) . P(II)$ (*d*) None of the above

(*v*) **Suppose $P(II) = \frac{2}{6}$, $P(I) = \frac{4}{6}$, $P(I \cap II) = \frac{2}{6}$, find $P(I \mid II)$.**

(*a*) 1 (*b*) 0 (*c*) $\frac{2}{6}$ (*d*) $\frac{4}{6}$

18. Case Study—The following image shows Big Bazaar in which seller sells x items at a price of ₹ $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5} + 500\right)$.

Answer the following questions:

(*i*) **Profit function in the form of x is represented as:**

(*a*) $\frac{x^2}{100} + \frac{24x}{5} - 500$ (*b*) $\frac{x^2}{100} - \frac{24x}{5} + 500$ (*c*) $\frac{24x}{5} + \frac{x^2}{100} + 500$ (*d*) $\frac{24x}{5} - \frac{x^2}{100} - 500$

(*ii*) **If the owner wants to maximize the profit how many items should be required to sell by him?**

(*a*) 120 items (*b*) 240 items (*c*) 200 items (*d*) 100 items

(*iii*) **If the total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.**

(*a*) 208 (*b*) 280 (*c*) 802 (*d*) 820

(*iv*) **Find the intervals in which the function f is given by $f(x) = x^2 - 4x + 6$ is strictly increasing.**

(*a*) $(-2, \infty)$ (*b*) $(-\infty, -2)$ (*c*) $(2, \infty)$ (*d*) $(-\infty, 2)$

(*v*) **Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.**

(*a*) 674 (*b*) 764 (*c*) 476 (*d*) 746

PART B

Section III

19. Simplify: $\cot^{-1} \frac{1}{\sqrt{x^2 - 1}}$ for $x < -1$.

20. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Or

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}.

21. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = O$. Using this result calculate A^3 also.

22. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

23. Evaluate : $\int \frac{x^3 + x + 1}{x^2 - 1} dx$.

Or

Evaluate : $\int e^x \frac{(1 - \sin x)}{(1 - \cos x)} dx$.

24. Compute, using integration, the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

25. Find the particular solution of the differential equation :
$(x - \sin y)\, dy + (\tan y)\, dx = 0$, given that $y = 0$ when $x = 0$

26. Find a unit vector perpendicular to the plane of triangle ABC where the vertices are A(3, –1, 2), B (1, –1, –3) and C (4, –3, 1).

27. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection.

28. There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. Write the probability distribution for the selected persons who are non-violent.

_Or_

A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Show that the relation R in the set $A = \{x : x \in Z, 0 \le x \le 12\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

30. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

31. Show that the function $g(x) = |x-2|, x \in R$, is continuous but not differentiable at $x = 2$.

_Or_

Differentiate $\log(x^{\sin x} + \cot^2 x)$ with respect to x.

32. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub-intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

33. Evaluate: $\int_{-1}^{1} \frac{x+|x|+1}{x^2+2|x|+1} dx$.

34. Using integration find the area of the region $\left[(x, y) : x^2 + y^2 \le 1 \le x + \frac{y}{2}, x, y \in R\right]$.

_Or_

Find the area of the region $\{(x, y) : y \ge x^2 \text{ and } y = |x|\}$.

35. Find the particular solution of the differential equation $\cos x\, dy = \sin x (\cos x - 2y)\, dx$, given that $y = 0$ when $x = \frac{\pi}{3}$.

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Using matrices, solve the following system of equations:

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4, \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2, x \ne 0, y \ne 0, z \ne 0.$

_Or_

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -2 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations:

$3x + 4y + 7z = 14, 2x - y + 3z = 4, x + 2y - 3z = 0.$

37. Find the equation of the plane through the points A(1, 1, 0), B(1, 2, 1), and C(–2, 2, –1) and hence find the distance between the plane and the line $\frac{x-6}{3} = \frac{y-3}{-1} = \frac{z+2}{1}$.

_Or_

Find the vector and Cartesian equations of the plane containing the two lines

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \text{ and } \vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$$

38. Solve the LLP graphically and find the maximum profit.

Maximize Profit, $Z = 200x + 20y$

Subject to the constraints: $3x + y \le 600; x + y \le 300; x - y \le 100; x, y \ge 0.$

_Or_

Solve the LLP graphically.

Minimize, $Z = 2x + 3y$

Subject to the constraints: $2x + 3y \ge 6; x - y \ge 0; 2x + y \le 8; x, y \ge 0.$

Answer Sheet

S A M P L E P A P E R

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MATHEMATICS

1. The smallest equivalence relation R on Set A = {1, 2, 3} is **R = {(1, 1), (2, 2), (3, 3)}**

Or

$x \le y^2$ as $8 \le (3)^2$ and $3 \le (2)^2$

$8 \nleq (2)^2$

$(8, 3) \in R$ and $(3, 2) \in R$

But $(8, 2) \notin R$

$\therefore$ **Relation R is not transitive**

2. $A = \{a, b, c\}$

The number of all one-one functions from the set A are 3! = **6**

(i) $f(1) = 1$, $f(2) = 2$, $f(3) = 3$
(ii) $f(1) = 1$, $f(2) = 3$, $f(3) = 2$
(iii) $f(1) = 2$, $f(2) = 3$, $f(3) = 1$
(iv) $f(1) = 2$, $f(2) = 1$, $f(3) = 3$
(v) $f(1) = 3$, $f(2) = 2$, $f(3) = 1$
(vi) $f(1) = 3$, $f(2) = 1$, $f(3) = 2$

3. Equivalence class of [(1, 3)] is given by set of ordered pair $(a, b) \in A \times A$ such that

$(1, 3)\ R\ (a, b) \quad \Rightarrow 1 + b = 3 + a \quad \therefore$ **[(1, 3)] = {(1, 3), (2, 4)}**

Or

For one-one

Let $x_1, x_2 \in N$ such that $f(x_1) = f(x_2)$ $\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow x_1^2 + x_1 + 1 - x_2^2 - x_2 - 1 = 0$ $\Rightarrow (x_1^2 - x_2^2) + x_1 - x_2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) + (x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 - x_2 = 0$ $[\because x_1 + x_2 + 1 \ne 0$ for any N

$\Rightarrow x_1 = x_2$ $\therefore$ f is one-one function

For onto

Clearly $f(x) = x^2 + x + 1 \ge 3$ for $x \in \mathbf{N}$

But $f(x)$ does not assume values 1 and 2

$\therefore$ $f: \mathbf{N} \to \mathbf{N}$ is not onto function

Hence the function is one-one.

4. $|A| = 20 - 42 = -22$

$A.(adj.A) = |A|.I_2$

$$= -22\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{-22} & \mathbf{0} \\ \mathbf{0} & \mathbf{-22} \end{bmatrix}$$

5. Let $A = \begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$

A is skew symmetric

$\therefore \quad A' = -A$

$$\begin{bmatrix} 2k+3 & -4 & -5 \\ 4 & 0 & 6 \\ 5 & -6 & -2k-3 \end{bmatrix} = \begin{bmatrix} -2k-3 & -4 & -5 \\ 4 & 0 & 6 \\ 5 & -6 & 2k+3 \end{bmatrix}$$

$\therefore \quad 2k + 3 = -2k - 3$

$4k = -6 \qquad \therefore\ k = -\frac{6}{4} = \mathbf{-\frac{3}{2}}$

Or

$$\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = \frac{1}{2}$$

$\Rightarrow \quad \sin\alpha\sin\beta - \cos\alpha\cos\beta = \frac{1}{2} \qquad \Rightarrow \quad -(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = \frac{1}{2}$

$\Rightarrow \quad \cos(\alpha+\beta) = -\frac{1}{2} \qquad \Rightarrow \quad \cos(\alpha+\beta) = -\cos\frac{\pi}{3}$

$\Rightarrow \quad \cos(\alpha+\beta) = \cos\left(\pi - \frac{\pi}{3}\right) \qquad [\because -\cos\theta = \cos(\pi - \theta)$

$\Rightarrow \quad \cos(\alpha+\beta) = \cos\left(\frac{2\pi}{3}\right) \qquad \therefore \quad \alpha + \beta = \frac{2\pi}{3}$ **or 120°**

6. $|adj\ A| = 225 \qquad \Rightarrow |A|^{3-1} = (15)^2 \qquad \Rightarrow |A| = 15$

$\therefore \quad \mathbf{|A'| = |A| = 15}$

7. $\int_0^1 (3x^2 + 2x + k)\,dx = 0$

$\left[\frac{3x^3}{3} + \frac{2x^2}{2} + kx\right]_0^1 = 0$

$\left[x^3 + x^2 + kx\right]_0^1 = 0$

$[1^3 + 1^2 + k(1)] - [0 + 0 + 0] = 0$

$1 + 1 + k = 0 \qquad \therefore\ \mathbf{k = -2}$

Or

$$\int \cot x\,(\operatorname{cosec} x - 1)\,e^x\,dx = \int [\operatorname{cosec} x \,.\, \cot x + (-\cot x)]\,e^x\,dx$$

$$= \int [f'(x) + f(x)]\,e^x\,dx$$

$= e^x f(x) + C \qquad$ **...where $[f(x) = -\cot x,\ f'(x) = \operatorname{cosec} x . \cot x$**

$= \mathbf{-e^x \,.\, \cot x + C}$

8. $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$

Let $p = e^x$
$\therefore \quad dp = e^x\,dx$
when $x = 1,\ p = e$
when $x = 0,\ p = 1$

$= \int_1^e \frac{dp}{1+p^2} = \left[\tan^{-1}(p)\right]_1^e$

$= \tan^{-1}(e) - \tan^{-1}(1) \Rightarrow \mathbf{\tan^{-1}(e) - \frac{\pi}{4}}$

9. Order, $m = 2$; Degree, $n = 2$

$\therefore \quad \boldsymbol{m + n} = 2 + 2 = \mathbf{4}$

Or

$\sec x \dfrac{dy}{dx} - y = \sin x$

Dividing both sides by $\sec x$, we get

$$\frac{dy}{dx} - \frac{y}{\sec x} = \frac{\sin x}{\sec x}$$

On comparing with $\dfrac{dy}{dx} + \text{P}y = \text{Q}$

$\therefore \quad \text{P} = \dfrac{-1}{\sec x} = -\cos x,\ \text{Q} = \dfrac{\sin x}{\sec x} = \sin x \cos x$

Integrating factor, IF $= e^{\int \text{P}\,dx} = e^{\int -\cos x\,dx} = e^{-\sin x}$

10. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow \quad a = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow \quad a^2 = x^2 + y^2 + z^2$ ...(*i*) **[Squaring both sides**

Here, $\vec{a} \times \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i} = -y\hat{k} + z\hat{j}$

and $|\vec{a} \times \hat{i}|^2 = y^2 + z^2$ ...(*ii*) $\quad ...\left[\begin{array}{l} \because \hat{i} \times \hat{i} = \mathbf{0} \\ \because \hat{j} \times \hat{i} = -\hat{k} \\ \because \hat{k} \times \hat{i} = \hat{j} \end{array}\right.$

Similarly, $|\vec{a} \times \hat{j}|^2 = x^2 + z^2$ ...(*iii*)

$|\vec{a} \times \hat{k}|^2 = x^2 + y^2$ ...(*iv*)

Now, $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$

$= (y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2)$ ...**[From (*ii*), (*iii*) and (*iv*)**

$= 2(x^2 + y^2 + z^2) = \mathbf{2a^2}$ ...**[From (*i*)**

11. $|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2$ $\quad [\because |\vec{m}|^2 = |\vec{m}|^2$

$= (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b}$

$= |\vec{a}|^2 + 0 + 0 + |\vec{b}|^2$

$|\vec{a} + \vec{b}|^2 = 1 + 1 = 2$

$\therefore \quad \mathbf{|\vec{a} + \vec{b}| = \sqrt{2}}$

$\left[\begin{array}{l} \text{Angle between} \\ \vec{a} \text{ and } \vec{b} \text{ is } 90° \ (120° - 30°) \\ \therefore \vec{a} \perp \vec{b} \\ \therefore \vec{a}.\vec{b} = 0 = \vec{b}.\vec{a} \end{array}\right.$

12. $\vec{a} \times \vec{b}$ is a unit vector

$\therefore \quad |\vec{a} \times \vec{b}| = 1, \quad |\vec{a}| = 3, \quad |\vec{b}| = \dfrac{2}{3}$

$\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}| \times |\vec{b}|} = \dfrac{1}{3\left(\frac{2}{3}\right)} = \dfrac{1}{2} \qquad \therefore \ \boldsymbol{\theta = \frac{\pi}{6}}$

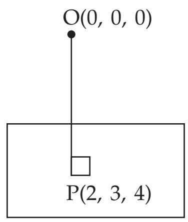
13. For each point A(a, b, c) on the x-axis is B(a, 0, 0)

$\therefore$ Distance of A from x-axis

$$AB = \sqrt{(a-a)^2 + (0-b)^2 + (0-c)^2}$$

$$= \sqrt{0 + b^2 + c^2} = \sqrt{\mathbf{b^2 + c^2}} \ \textbf{units}$$

14. $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{n} = \overrightarrow{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$



So, the equation of plane is

$$(\vec{r} - \vec{a}).\vec{n} = 0 \Rightarrow \vec{r}.\vec{n} - \vec{a}.\vec{n} = 0$$

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$\Rightarrow$ $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k})(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\Rightarrow$ $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = 4 + 9 + 16$

$\therefore$ $\boldsymbol{\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = 29}$

15. Let p_1, p_2, p_3, p_4 be the chances of solving the problem by A, B, C and D respectively.

$$p_1 = \frac{1}{3}, \qquad p_2 = \frac{1}{4}, \qquad p_3 = \frac{1}{5}, \qquad p_4 = \frac{2}{3}$$

$$\therefore \quad q_1 = 1 - \frac{1}{3} = \frac{2}{3}, \quad q_2 = 1 - \frac{1}{4} = \frac{3}{4}, \quad q_3 = 1 - \frac{1}{5} = \frac{4}{5}, \quad q_4 = 1 - \frac{2}{3} = \frac{1}{3}$$

$\therefore$ **P (at most one of them will solve the problem)** = P(none of them) + P(one of them)

$$= q_1q_2q_3q_4 + (p_1q_2q_3q_4 + q_1p_2q_3q_4 + q_1q_2p_3q_4 + q_1q_2q_3p_4)$$

$$= \left(\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}\right) + \left(\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}\times\frac{1}{3}\right) + \left(\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}\times\frac{1}{3}\right) + \left(\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{2}{3}\right)$$

$$= \frac{24 + 12 + 8 + 6 + 48}{180} = \frac{98}{180} = \mathbf{\frac{49}{90}}$$

16. As we know, $P(\overline{A}/\overline{B}) = \dfrac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \dfrac{1 - P(A \cup B)}{P(\overline{B})}$

$$= \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)}$$

$$= \frac{1 - \left[\frac{2}{5} + \frac{1}{3} - \frac{1}{5}\right]}{1 - \frac{1}{3}} = \frac{\frac{15 - 6 - 5 + 3}{15}}{\frac{2}{3}} = \frac{7}{15} \times \frac{3}{2} = \mathbf{\frac{7}{10}}$$

17. ***(i)*** (*a*); Let E_1, E_2 and E_3 be the production by three machines.

$$P(E_1) = 30\% = \frac{30}{100} = \mathbf{\frac{3}{10}}, \ P(E_2) = 50\% = \frac{50}{100} = \mathbf{\frac{5}{10}}, \ P(E_3) = 20\% = \frac{20}{100} = \mathbf{\frac{2}{10}}$$

$\therefore$ Probability of production by three machines = $\mathbf{\frac{3}{10}, \frac{5}{10}, \frac{2}{10}}$

(ii) (*c*); Let E be the event of a defective item

$$P(E/E_1) = 2\% = \frac{2}{100}, \ P(E/E_2) = 5\% = \frac{5}{100}, \ P(E/E_3) = 3\% = \frac{3}{100}$$

$\therefore$ Probability of production of three machines if an item is picked and found be defective is $\mathbf{\frac{2}{100}, \frac{5}{100}, \frac{3}{100}}$.

(*iii*) (*b*); **Required probability, $P(E_3/E)$** $= \dfrac{P(E_3)P(E/E_3)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2) + P(E_3)P(E/E_3)}$

$$= \frac{\frac{2}{10} \times \frac{3}{100}}{\frac{3}{10} \times \frac{2}{100} + \frac{5}{10} \times \frac{5}{100} + \frac{2}{10} \times \frac{3}{100}}$$

$$= \frac{\frac{6}{1000}}{\frac{6}{1000} + \frac{25}{1000} + \frac{6}{1000}} = \frac{\frac{6}{1000}}{\frac{37}{1000}} = \mathbf{\frac{6}{37}}$$

(*iv*) (*a*); Machine I and Machine II are independent events when

$\therefore$ **$P(A \cap B) = P(A) . P(B)$**

(*v*) (*a*); Here $P(I/II) = \dfrac{P(I \cap II)}{P(II)} = \dfrac{2/6}{2/6} = \mathbf{1}$

18. (*i*) (*d*); Let $S(x)$ be the selling price of x items and let $C(x)$ be the cost price of x items.

$S(x) = \left(5 - \dfrac{x}{100}\right)x = 5x - \dfrac{x^2}{100}$ and $C(x) = \dfrac{x}{5} + 500$

Thus, the profit function $P(x)$ is given by $P(x) = S(x) - C(x)$

$$= 5x - \frac{x^2}{100} - \frac{x}{5} - 500 = \mathbf{\frac{24x}{5} - \frac{x^2}{100} - 500}$$

(*ii*) (*b*); We have, $P(x) = \dfrac{24x}{5} - \dfrac{x^2}{100} - 500$ **...[From point (*i*)**

Differentiating the both sides wrt. x, we have

$$P'(x) = \frac{24}{5} - \frac{x}{50}$$

when $P'(x) = 0$, $\dfrac{24}{5} - \dfrac{x}{50} = 0$

$\Rightarrow \dfrac{x}{50} = \dfrac{24}{5} \Rightarrow x = \dfrac{24 \times 50}{5} = 240$

Also, $P''(x) = \dfrac{-1}{50} < 0(-\text{ve})$ (maximum)

Hence, the owner can maximize his profits by selling 240 items.

(*iii*) (*a*); We have, $R(x) = 13x^2 + 26x + 15$

Differentiating the above w.r.t. x, we get

$R'(x) = 26x + 26$

As we know, $R'(x) = MR$

$\therefore$ Marginal Revenue (MR) $= 26x + 26$

$[MR]$ at $x = 7 = 26(7) + 26$

$= 26\,(7 + 1) = 26 \times 8 = \mathbf{208}$

(*iv*) (*c*); We have, $f(x) = x^2 - 4x + 6$

Differentiating the above w.r.t. x, we have

$f'(x) = 2x - 4$

$f'(x) = 0 \quad \Rightarrow 2x - 4 = 0 \quad \therefore \mathbf{x = \frac{4}{2} = 2}$

Intervals are $(-\infty, 2)$ and $(2, \infty)$.

Therefore, in the interval **$(2, \infty)$**, $f'(x) > 0$ and function is strictly increasing.

(v) (c); We have, $y = 3x^4 - 4x$

Differentiating the above w.r.t. r, we have

$$\frac{dy}{dx} = 12x^3 - 4$$

$$\therefore \left.\frac{dy}{dx}\right]_{x=4} = 12(4)^3 - 4 = \mathbf{764}$$

19. $$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta - 1}}\right) \quad \ldots\left[\begin{array}{l}\text{Let } x = \sec\theta \text{ for } x < -1, \\ \Rightarrow \theta = \sec^{-1} x \\ \text{where } \frac{\pi}{2} < \theta < \pi\end{array}\right.$$

$$= \cot^{-1}\left(\frac{1}{-\tan\theta}\right) \quad \ldots[\because \sec^2\theta - 1 = \tan^2\theta$$

$$= \cot^{-1}(-\cot\theta) = \cot[\cot(\pi - \theta)]$$

$$= \pi - \theta$$

$$= \pi - \mathbf{sec^{-1}}\, x \text{ as } 0 < \pi - \theta < \frac{\pi}{2}.$$

20. A is a skew-symmetric matrix

$A^t = -A$

$\Rightarrow$ $|A^t| = |-A|$

$\Rightarrow$ $|A^t| = (-1)^n |A|$

$\Rightarrow$ $|A^t| = (-1)^3 |A|$ ...[$\because n = 3$

$\Rightarrow$ $|A| = -|A|$ ...[$\because |A^t| = |A|$

$\Rightarrow$ $|A| + |A| = 0$

$\Rightarrow$ $2|A| = 0$ $\therefore$ $\mathbf{|A| = 0}$

Or

L.H.S. $A^2 - 5A - 14I$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \textbf{R.H.S.}$$

$A^2 - 5A - 14I = O$ $\Rightarrow$ $A^2 - 5A = 14I$ $\left[\begin{array}{l}\because \mathbf{IA^{-1} = A^{-1}} \\ \mathbf{AA^{-1} = I} \\ \mathbf{A^2A^{-1} = AI = A}\end{array}\right.$

$(A^2 - 5A).A^{-1} = 14IA^{-1}$ $\Rightarrow$ $A^2.A^{-1} - 5AA^{-1} = 14A^{-1}$

$A - 5I = 14A^{-1}$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 14A^{-1} \quad \Rightarrow \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 14A^{-1}$$

$$\Rightarrow 14A^{-1} = \begin{bmatrix} 3-5 & -5-0 \\ -4-0 & 2-5 \end{bmatrix} \quad \therefore \mathbf{A^{-1} = \frac{1}{14}\begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix}}.$$

21. $$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} \quad \ldots(i)$$

L.H.S. $= A^2 - 4A + 7I$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ **[From (*i*)**

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \textbf{R.H.S.}$$

From (*i*),

$A^2 - 4A + 7I = O$ **[Proved above**

$A^2 = 4A - 7I$ ...(*ii*)

$A^2 . A = (4A - 7I)A$ **[Post multiply by A**

$A^3 = 4A^2 - 7IA$

$A^3 = 4(4A - 7I) - 7A$ **[$\because$ IA = A] [From (*iii*)**

$A^3 = 16A - 28I - 7A$

$A^3 = 9A - 28I$

$$A^3 = 9\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 28\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow A^3 = \begin{bmatrix} 18 & 27 \\ -9 & 18 \end{bmatrix} - \begin{bmatrix} 28 & 0 \\ 0 & 28 \end{bmatrix}$$

$$\therefore \quad \mathbf{A^3} = \begin{bmatrix} 18-28 & 27-0 \\ -9-0 & 18-28 \end{bmatrix} = \begin{bmatrix} \mathbf{-10} & \mathbf{27} \\ \mathbf{-9} & \mathbf{-10} \end{bmatrix}$$

22. $xy = a^2 \Rightarrow y = \dfrac{a^2}{x}$ ...(*i*)

$x^2 + y^2 = 2a^2$ ...(*ii*)

$$\Rightarrow \quad x^2 + \left(\frac{a^2}{x}\right)^2 = 2a^2$$ **[From (*i*)**

$$\Rightarrow \quad \frac{x^4 + a^4}{x^2} = 2a^2 \quad \Rightarrow x^4 + a^4 = 2a^2x^2 \quad \Rightarrow x^4 - 2a^2x^2 + a^4 = 0$$

$$\Rightarrow \quad (x^2)^2 - 2(x^2)(a^2) + (a^2)^2 = 0 \quad \Rightarrow (x^2 - a^2)^2 = 0$$

$$\Rightarrow \quad (x^2 - a^2) = 0 \quad \Rightarrow x^2 = a^2 \quad \therefore \ x = \pm a$$

From (*i*), when $x = a$

$$y = \frac{a^2}{a} = a$$

Point A(a, a)

From (*i*), when $x = -a$

$$y = \frac{a^2}{-a} = -a$$

Point B($-a$, $-a$)

Thus, the two curves intersect at A(a, a) and B($-a$, $-a$).

From (*i*), $y = \dfrac{a^2}{x}$

Differentiating both sides w.r.t. x, we get $\dfrac{dy}{dx} = \dfrac{-a^2}{x^2}$

$\dfrac{dy}{dx}$ at A(a, a) $= \dfrac{-a^2}{a^2} = -1$ ...(*iii*)

$\dfrac{dy}{dx}$ at B($-a$, $-a$) $= \dfrac{-a^2}{a^2} = -1$ ...(*iv*)

From (*ii*), $x^2 + y^2 = 2a^2$

Differentiating both sides w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0 \quad \Rightarrow 2y\frac{dy}{dx} = -2x \quad \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{dy}{dx} \text{ at A}(a, a) = \frac{-a}{a} = -1 \qquad \ldots(v)$$

$$\frac{dy}{dx} \text{ at B}(-a, -a) = \frac{-(-a)}{-a} = -1 \qquad \ldots(vi)$$

From (*iii*) and (*v*), $\underset{\textit{Curve I}}{\frac{dy}{dx} \text{ at A}} = \underset{\textit{Curve II}}{\frac{dy}{dx} \text{ at A}}$

So, the two curves touch each other at A.

From (*iv*) and (*vi*), $\underset{\textit{Curve I}}{\frac{dy}{dx} \text{ at B}} = \underset{\textit{Curve II}}{\frac{dy}{dx} \text{ at B}}$

So, the two curves touch each other at B.

$\therefore$ **The two curves touch each other at A as well as at B.**

23.
$$\int \frac{x^3+x+1}{x^2-1}\,dx = \int \left(x + \frac{2x+1}{x^2-1}\right) dx$$

$$\left[\begin{array}{r|l} x^2-1 & x^3+x+1 \\ & \underline{\overset{-}{+}x^3 \overset{+}{-} x} \\ & 2x+1 \end{array}\right.$$

$$= \int x\,dx + \int \frac{2x}{x^2-1}\,dx + \int \frac{1}{x^2-1}\,dx$$

$$= \frac{x^2}{2} + \int \frac{dp}{p} + \int \frac{dx}{x^2-1^2} \qquad \textbf{[Let } p = x^2-1,\ dp = 2x\,dx$$

$$= \frac{x^2}{2} + \log|p| + \frac{1}{2(1)}\log\left|\frac{x-1}{x+1}\right| + c \qquad \left[\because \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right.$$

$$= \frac{x^2}{2} + \log|x^2-1| + \frac{1}{2}(\log|x-1| - \log|x+1|) + c$$

$$= \frac{x^2}{2} + \log|x+1| + \log|x-1| + \frac{1}{2}\log|x-1| - \frac{1}{2}\log|x+1| + c$$

$$= \mathbf{\frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + c}$$

Or

$$\int e^x \frac{(1-\sin x)}{(1-\cos x)}\,dx = \int e^x \frac{\left(1 - 2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{2\sin^2\frac{x}{2}}\,dx$$

$$= \int e^x \left(\frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$$

$$= \int e^x \left[\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} + \left(-\cot\frac{x}{2}\right)\right] dx \qquad \ldots\left[\begin{array}{l}\because \textbf{ where } f(x) = -\cot\frac{x}{2} \\ f'(x) = \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}dx\end{array}\right.$$

$$= \int e^x\,[f'(x) + f(x)]\,dx$$

$$= e^x f(x) + c = \mathbf{-e^x \cdot \cot\frac{x}{2} + c}$$

24.

$x + 2y = 2$

$\Rightarrow x = 2 - 2y$

x	0	2
y	1	0

$y - x = 1$

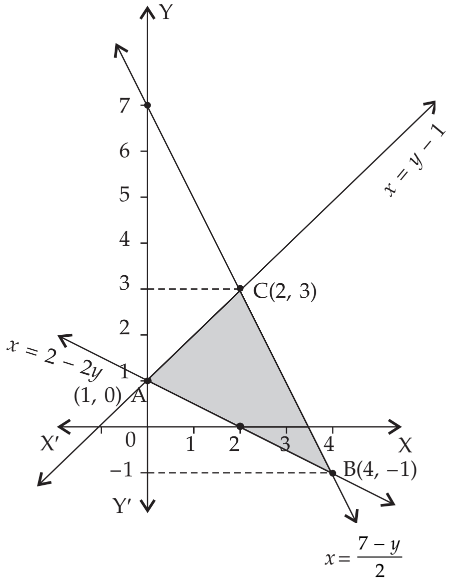
$\Rightarrow y - 1 = x$

x	0	2
y	1	3

$2x + y = 7$

$\Rightarrow x = \dfrac{7-y}{2}$

x	2	4
y	3	-1



$\therefore$ Points of intersection are A(1, 0), B(4, –1) and C(2, 3).

Area of the shaded region $= \int_{-1}^{3} \frac{7-y}{2}\,dy - \int_{-1}^{1} (2-2y)\,dy - \int_{1}^{3} (y-1)\,dy$

$\left[\because \text{ Shaded Area} = \int_a^b x\,dy \text{ where } a, b \text{ from Y-axis}\right.$

$$= \frac{1}{2}\left[7y - \frac{y^2}{2}\right]_{-1}^{3} - \left[2y - \frac{2y^2}{2}\right]_{-1}^{1} - \left[\frac{y^2}{2} - y\right]_{1}^{3}$$

$$= \frac{1}{2}\left[\left(7(3) - \frac{9}{2}\right) - \left(-7 - \frac{1}{2}\right)\right] - [(2-1) - (-2-1)] - \left[\left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right]$$

$= 12 - 4 - 2 =$ **6 sq. units.**

25. $(x - \sin y)\,dy + (\tan y)\,dx = 0$

$\Rightarrow \quad \tan y\,dx = -(x - \sin y)\,dy \qquad \Rightarrow \quad \frac{dx}{dy} = \frac{-x + \sin y}{\tan y}$

$\Rightarrow \quad \frac{dx}{dy} = \frac{-x}{\tan y} + \frac{\sin y}{\tan y} \qquad \Rightarrow \quad \frac{dx}{dy} + x.\cot y = \cos y$

Here 'P' $= \cot y$, Q $= \cos y$

$\text{I.F.} = e^{\int P\,dy} = e^{\int \cot y\,dy} = e^{\log \sin y} = \sin y$

$\therefore$ The solution is $x\,(\text{I.F.}) = \int Q(\text{I.F.})\,dy$

$x(\sin y) = \int \cos y \sin y\,dy$

$= \frac{1}{2}\int 2\sin y \cos y\,dy = \frac{1}{2}\int \sin 2y\,dy$

$x(\sin y) = \frac{-\cos 2y}{4} + C \qquad \ldots(i)$

It is given that $y = 0$, when $x = 0$

$0.\sin 0 = \frac{-\cos 0}{4} + C \qquad \Rightarrow \quad \frac{1}{4} = C$

Putting the value of C in (i),

$$x \sin y = \frac{-\cos 2y}{4} + \frac{1}{4} \qquad \Rightarrow \quad x(\sin y) = \frac{1-\cos 2y}{4} = \frac{1}{2}\sin^2 y$$

$\Rightarrow \quad x = \frac{1}{2}\sin y$

$\Rightarrow$ **$2x = \sin y$ is the required solution.**

26. Vertices of ΔABC are A(3, –1, 2), B(1, –1, –3) and C(4, –3, 1). **...[Given**

$\overrightarrow{AB} \times \overrightarrow{AC}$ is perpendicular to the plane of ΔABC.

$$\overrightarrow{AB} = (1-3)\hat{i} + (-1+1)\hat{j} + (-3-2)\hat{k} = -2\hat{i} + 0\hat{j} - 5\hat{k}$$

$$\overrightarrow{AC} = (4-3)\hat{i} + (-3+1)\hat{j} + (1-2)\hat{k} = \hat{i} - 2\hat{j} - \hat{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}(0-10) - \hat{j}(2+5) + \hat{k}(4-0)$$

$$= -10\hat{i} - 7\hat{j} + 4\hat{k}$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{100+49+16} = \sqrt{165}$$

$\therefore$ Required unit vector $\perp$ to the plane of ΔABC $= \frac{(\overrightarrow{AB} \times \overrightarrow{AC})}{|\overrightarrow{AB} \times \overrightarrow{AC}|} = \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}$

$$= -\frac{\mathbf{10}}{\sqrt{\mathbf{165}}}\hat{i} - \frac{\mathbf{7}}{\sqrt{\mathbf{165}}}\hat{j} + \frac{\mathbf{4}}{\sqrt{\mathbf{165}}}\hat{k}$$

27. **Part I :** $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ ...(i)

$$= (1+3\lambda)\hat{i} + (1-\lambda)\hat{j} - \hat{k}$$

Let P($1 + 3\lambda$, $1 - \lambda$, –1) be any point on the line (i)

$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ ...(ii)

$$= (4+2\mu)\hat{i} + (-1+3\mu)\hat{k}$$

Let Q($4 + 2\mu$, 0, $-1 + 3\mu$) be any point on the line (ii)

If given two lines intersect,

P and Q must coincide for some λ and μ.

$\therefore \quad 1 + 3\lambda = 4 + 2\mu$ ...(1) | $1 - \lambda = 0$, $\lambda = 1$ | $-1 = -1 + 3\mu$, $0 = 3\mu \Rightarrow 0 = \mu$

Putting the values of λ and μ in (1), we have

$1 + 3(1) = 4 + 2(0) \quad \Rightarrow \quad 4 = 4$, which is true.

Hence, the given two lines are intersecting.

Part II : For point of intersection

1st method		**2nd method**
$(1 + 3\lambda, 1 - \lambda, -1)$		$(4 + 2\mu, 0, -1 + 3\mu)$
$= (1 + 3, 1 - 1, -1)$		$(4 + 0, 0, -1 + 0)$
= P(4, 0, –1)	or	**Q(4, 0, –1)**

28. Let x_i denote the number of non-violent persons out of selected two and therefore, x_i can take values 0, 1, and 2.

Non-violent = 20 $\quad \therefore$ Violent patriotism = 50 – 20 = 30

$$P(x_i = 0) = \frac{^{30}C_2}{^{50}C_2} = \frac{30 \times 29}{50 \times 49} = \frac{87}{245}$$

$$P(x_i = 1) = \frac{^{20}C_1 \times {}^{30}C_1}{^{50}C_2} = \frac{\frac{20}{1} \times \frac{30}{1}}{\frac{50 \times 49}{2 \times 1}} = \frac{120}{245}$$

$$P(x_i = 2) = \frac{^{20}C_2}{^{50}C_2} = \frac{20 \times 19}{50 \times 49} = \frac{38}{245}$$

Probability distribution is

x_i	0	1	2
$P(x_i)$	$\frac{87}{245}$	$\frac{120}{245}$	$\frac{38}{245}$

Mean $= \Sigma P_i x_i = \left(0 \times \frac{87}{245}\right) + \left(1 \times \frac{120}{245}\right) + 2 \times \frac{38}{245}$

$$= \frac{120}{245} + \frac{76}{245} = \mathbf{\frac{196}{245}}$$

Or

Number obtained is even *i.e.*, A = 2, 4, 6
Number obtained is red *i.e.*, B = 1, 2, 3

$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2}, \quad P(B) = \frac{3}{6} = \frac{1}{2}$

$P(A \cap B) = P$ (even red number)

$= P$ (number 2) $= \frac{1}{6}$

$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\Rightarrow \quad P(A) \times P(B) \neq P(A \cap B)$

Hence, A and B are not independent events.

29. $A = \{x : x \in Z, 0 \le x \le 12\}$

$R = \{(a, b) : |a - b|$ is divisible by 4$\}$

Part I: R is reflexive, as for any $a \in A$

$|a - a| = 0$, is divisible by 4

$(a, a) \in R \quad \ldots(i) \quad \therefore \quad$ R is reflexive

R is symmetric as for any $(a, b) \in R$,

$|a - b|$ is divisible by 4

$|a - b| = 4k, \quad k \in Z \quad \Rightarrow \quad |b - a| = 4k, \quad k \in R$

$(b, a) \in R \quad \ldots(ii) \quad \therefore \quad$ R is symmetric

R is transitive. Let $a, b, c \in A$

$(a, b) \in R$

$(b, c) \in R$

$|a - b|$ is divisible by 4

Let $|a - b| = 4k, k \in Z \quad \therefore \quad (a - b) = \pm 4k$

$|b - c|$ is divisible by 4,

Let $|b - c| = 4m, m \in Z \quad \therefore \quad (b - c) = \pm 4m$

$a - c = (a - b) + (b - c) = \pm 4k \pm 4m = \pm 4(k + m)$

$\therefore \quad a - c$ is divisible by 4.

$|a-c|$ is divisible by 4. $\therefore$ $(a, c) \in R$

$\therefore$ R is transitive. ...(*iii*)

From (*i*), (*ii*) and (*iii*), **R is an equivalence relation.**

Part II: Let x be an element of A such that $(x, 1) \in R$

$|x-1|$ is divisible by 4

$|x-1| = 0, 4, 8, 12$ ...[$\because$ $0 \le x \le 12$ (given)]

$x - 1 = 0, 4, 8, 12$

$x = 1, 5, 9, 13 \rightarrow 13$ is rejected

$\therefore$ **Set of all elements of A related to 1 is {1, 5, 9}**

30. $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Let $x = \sin A \Rightarrow A = \sin^{-1} x$ and $y = \sin B \Rightarrow B = \sin^{-1} y$ ...(*i*)

$\therefore$ $\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow$ $\cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow$ $2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = a\, 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow$ $\cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow$ $\frac{1}{a} = \tan\left(\frac{A-B}{2}\right)$

$\Rightarrow$ $\frac{A-B}{2} = \tan^{-1}\left(\frac{1}{a}\right)$ $\Rightarrow$ $A - B = 2\tan^{-1}\left(\frac{1}{a}\right)$

$\sin^{-1} x - \sin^{-1} y = 2\tan^{-1}\left(\frac{1}{a}\right)$ **[From (*i*)**

Differentiating both sides w.r.t. x, we get

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow$ $-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

$\therefore$ $\boldsymbol{\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}}$

31. $g(x) = |x-2| = \begin{cases} -(x-2), & x < 2 \\ (x-2), & x \ge 2 \end{cases}$

For continuity at $x = 2$

L.H.L. $= \lim_{x\to 2^-} g(x)$

$= \lim_{x\to 2^-} -(x-2)$

$= \lim_{h\to 0} -[2-h-2]$ ...**[Putting $x = 2-h, h > 0$]**

$= \lim_{h\to 0} h = 0$

R.H.L. $= \lim_{x\to 2^+} g(x)$

$= \lim_{x\to 2^+} (x+2)$

$= \lim_{h\to 0} [2+h-2]$ ...**[Putting $x = 2+h, h > 0$]**

$= \lim_{h\to 0} h = 0$

At $x = 2$, $g(x) = (x-2)$,

$g(2) = 2 - 2 = 0$

L.H.L. = R.H.L. = $g(2)$ **(Hence Proved)**

$\therefore$ **$g(x)$ is continuous at $x = 2$**

For differentiability at $x = 2$

L.H.D. $= \lim_{x\to 2^-} \frac{g(x)-g(2)}{x-2}$

R.H.D. $= \lim_{x\to 2^+} \frac{g(x)-g(2)}{x-2}$

$$= \lim_{x \to 2^-} \frac{-(x-2)-0}{(x-2)}$$
$$= \lim_{x \to 2^-} (-1)$$
$$= -1$$

$$= \lim_{x \to 2^+} \frac{(x-2)-0}{(x-2)}$$
$$= \lim_{x \to 2^+} 1$$
$$= 1$$

L.H.D. $\neq$ R.H.D.

$\therefore$ **$g(x)$ is not differentiable at $x = 2$.**

Or

Let A $= x^{\sin x}$

Taking log on both sides

$$\log A = \sin x \,.\, \log x$$

Differentiating both sides w.r.t. x

$$\frac{1}{A}\frac{dA}{dx} = \sin x \,.\, \frac{1}{x} + \log x . \cos x$$

$$\frac{dA}{dx} = A\left(\sin x \,.\, \frac{1}{x} + \log x . \cos x\right)$$

$$\frac{d}{dx}(x^{\sin x}) = x^{\sin x}\left(\frac{\sin x}{x} + \log x . \cos x\right) \quad \ldots(i)$$

Let $y = \log (x^{\sin x} + \cot^2 x)$

Differentiating both sides w.r.t. x, we have

$$\frac{dy}{dx} = \frac{1}{(x^{\sin x} + \cot^2 x)} \times \frac{d}{dx}(x^{\sin x} + \cot^2 x)$$

$$= \frac{1}{(x^{\sin x} + \cot^2 x)} \times \left[\frac{d}{dx}(x^{\sin x}) + \frac{d}{dx}(\cot^2 x)\right]$$

$$= \frac{\mathbf{1}}{(x^{\sin x} + \cot^2 x)} \cdot \left[x^{\sin x}\left(\frac{\sin x}{x} + \log x . \cos x\right) - 2 \cot x . \operatorname{cosec}^2 x\right] \quad \textbf{[From } (i)$$

32. $f(x) = \sin^4 x + \cos^4 x,\ \left(0, \frac{\pi}{2}\right)$

Differentiating both sides w.r.t. x, we have

$$f'(x) = 4 \sin^3 x \,.\, \cos x + 4 \cos^3 x \,.\, (-\sin x)$$
$$= 4 \sin x \cos x \,(\sin^2 x - \cos^2 x)$$
$$= -2 \,.\, 2 \sin x \cos x \,(\cos^2 x - \sin^2 x)$$
$$= -2 \,.\, \sin 2x \,.\, \cos 2x = -\sin 4x$$

When $f'(x) = 0$

$-\sin 4x = 0 \quad \Rightarrow \quad \sin 4x = 0$

$\Rightarrow \quad 4x = 0, \pi, 2\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots$

Intervals	*Checking point*	*Sign of $-\sin 4x$*	*Sign of $f'(x)$*	*Nature of $f(x)$*
$\left(0, \frac{\pi}{4}\right)$	$x = \frac{\pi}{6}$	–ve	≤ 0	decreasing
$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	$x = \frac{\pi}{2}$	+ve	≥ 0	increasing

So, $f(x)$ is decreasing on $\left(0, \frac{\pi}{4}\right)$ and increasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

33. $$I = \int_{-1}^{1} \frac{x + |x| + 1}{x^2 + 2|x| + 1} dx$$

$$I = \int_{-1}^{1} \frac{x}{x^2 + 2|x| + 1} dx + \int_{-1}^{1} \frac{|x| + 1}{x^2 + 2|x| + 1} dx \quad \ldots(i)$$

Now, $I_1 = \int_{-1}^{1} \frac{x}{x^2 + 2|x| + 1} dx$

Let $f(x) = \frac{x}{x^2 + 2|x| + 1}$

$$f(-x) = \frac{-x}{(-x)^2 + 2|-x| + 1} = \frac{-x}{x^2 + 2|x| + 1} = -f(x)$$

$\because$ $f(x)$ is an odd function.

Hence, $I_1 = 0 \quad \ldots(ii)$

Also, $I_2 = \int_{-1}^{1} \frac{|x| + 1}{x^2 + 2|x| + 1} dx$

Let $g(x) = \frac{|x| + 1}{x^2 + 2|x| + 1}$

$$\Rightarrow \quad g(-x) = \frac{|-x| + 1}{(-x)^2 + 2|-x| + 1} = \frac{|x| + 1}{x^2 + 2|x| + 1} = g(x)$$

$\because$ $g(x)$ is even function,

$$\therefore \quad I_2 = 2\int_0^1 \frac{x + 1}{x^2 + 2x + 1} = 2\int_0^1 \frac{1}{x + 1} dx$$

$$= 2[\log|x + 1|]_0^1 = 2 \log 2 - 2 \log 1$$

$$I_2 = 2[\log 2 - \log 1] = 2 \log 2 \quad \ldots(iii)$$

From (i), (ii) and (iii), we get

$\therefore$ $\mathbf{I = I_1 + I_2 = 0 + 2\log 2 = 2 \log 2}$

34. $x^2 + y^2 \leq 1$

Let $x^2 + y^2 = 1$

$y = \pm\sqrt{1 - x^2}$

x	0	± 1
y	± 1	0

$x + \frac{y}{2} \geq 1$

Let $x + \frac{y}{2} = 1$

$\frac{y}{2} = 1 - x$

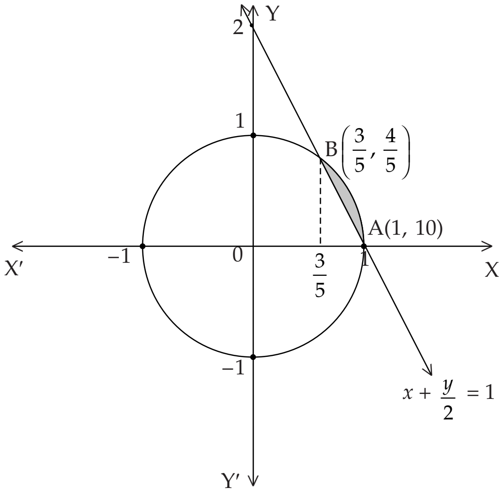
$y = 2(1 - x) = 2 - 2x \quad \ldots(i)$

x	0	1	0.5
y	2	0	1

For the point of intersection,

$x^2 + y^2 = 1$

$x^2 + (2 - 2x)^2 = 1 \quad$ **…[From (i)**



$\Rightarrow \quad x^2 + 4 + 4x^2 - 8x - 1 = 0$

$\Rightarrow \quad 5x^2 - 8x + 3 = 0 \qquad \Rightarrow 5x^2 - 5x - 3x + 3 = 0$

$\Rightarrow \quad 5x(x-1) - 3(x-1) = 0 \qquad \Rightarrow (x-1)(5x-3) = 0$

$\quad x - 1 = 0 \quad$ or $\quad 5x - 3 = 0$

$\Rightarrow \quad x = 1 \quad$ or $\quad x = \frac{3}{5}$

When $x = \frac{3}{5}, \quad y = 2 - 2\left(\frac{3}{5}\right) = \frac{10-6}{5} = \frac{4}{5}$ ...[From (*i*)]

$\therefore \quad$ Point B$\left(\frac{3}{5}, \frac{4}{5}\right)$

Area of the shaded region

$$= \int_{3/5}^{1} \sqrt{1^2 - x^2}\, dx - \int_{3/5}^{1} 2(1-x)\, dx \qquad \ldots\left[\because \int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right) + C\right]$$

$$= \frac{1}{2}\left[x\sqrt{1-x^2} + 1^2.\sin^{-1}\frac{x}{1}\right]_{3/5}^{1} - 2\left[x - \frac{x^2}{2}\right]_{3/5}^{1}$$

$$= \frac{1}{2}\left\{[1(0) + \sin^{-1}(1)] - \left[\frac{3}{5}\sqrt{1 - \frac{9}{25}} + \sin^{-1}\left(\frac{3}{5}\right)\right]\right\} - 2\left[\left(1 - \frac{1}{2}\right) - \left(\frac{3}{5} - \frac{1}{2} \times \frac{9}{25}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{3}{5} \times \frac{4}{5} - \sin^{-1}\frac{3}{5}\right] - 2\left[\frac{1}{2} - \frac{21}{50}\right]$$

$$= \frac{\pi}{4} - \frac{6}{25} - \frac{1}{2}\sin^{-1}\frac{3}{5} - 2\left[\frac{4}{50}\right]$$

$$= \frac{\pi}{4} - \frac{6}{25} - \frac{1}{2}\sin^{-1}\frac{3}{5} - \frac{4}{25}$$

$$= \frac{\pi}{4} - \frac{10}{25} - \frac{1}{2}\sin^{-1}\frac{3}{5} = \left(\frac{\pi}{4} - \frac{2}{5} - \frac{1}{2}\sin^{-1}\frac{3}{5}\right) \textbf{ sq. units}$$

Or

$y \geq x^2$; Let $y = x^2$

x	0	±1	±2
y	0	1	4

$y = |x|$

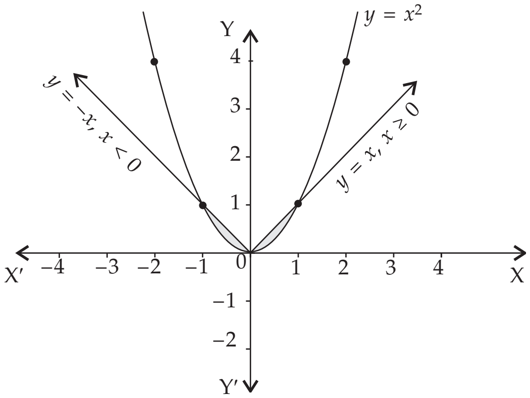
$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

$y = -x, x < 0$

x	-1	-2
y	1	2

$y = x, x \geq 0$

x	0	1	2
y	0	1	2



Area of shaded region = 2[Area of shaded region in I[st] quadrant]

$$= 2\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right] = 2\left\{\frac{1}{2}\left[x^2\right]_0^1 - \frac{1}{3}\left[x^3\right]_0^1\right\}$$

$$= 2\left[\frac{1}{2}(1-0) - \frac{1}{3}(1-0)\right]$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2\left(\frac{3-2}{6}\right) = \frac{\mathbf{1}}{\mathbf{3}}\ \textbf{sq. unit}$$

35. $\cos x\, dy = \sin x\,(\cos x - 2y)\, dx$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{\sin x \cos x}{\cos x} - \dfrac{2y \sin x}{\cos x}$ $\qquad \Rightarrow \quad \dfrac{dy}{dx} + 2y \tan x = \sin x$

Here 'P' = $2 \tan x$, 'Q' = $\sin x$

I.F = $e^{\int P\,dx} = e^{\int 2\tan x\,dx} = e^{2\log|\sec x|} = e^{\log(\sec^2 x)} = \sec^2 x$

Hence, the solution is

$y\,(\text{I.F.}) = \int Q(\text{I.F.})\,dx + C$ $\qquad \Rightarrow y\,(\sec^2 x) = \int \sin x \,.\, \sec^2 x\,dx + C$

$\Rightarrow \quad y\,(\sec^2 x) = \int \sec x \tan x\,dx + C$ $\qquad \Rightarrow y\,(\sec^2 x) = \sec x + C$ $\qquad \ldots(i)$

$\Rightarrow \quad 0 = \sec\dfrac{\pi}{3} + C$ $\qquad$ **...**$\left[y = 0, x = \dfrac{\pi}{3}\right.$

$\Rightarrow \quad 0 = 2 + C$ $\qquad \therefore\ C = -2$

Putting the value of C in (i), we have

$\Rightarrow \quad y(\sec^2 x) = \sec x - 2$ $\qquad \therefore\ y = \dfrac{\sec x}{\sec^2 x} - \dfrac{2}{\sec^2 x}$

Therefore, $y = \cos x - 2\cos^2 x$, which is the particular solution of the given differential equation.

36. Given equations can be written as

$$AX = B \Rightarrow X = A^{-1} B \quad \ldots(i)$$

$$\ldots\text{where}\left[A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}\right.$$

$|A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0 \quad \therefore \quad A^{-1}$ exists.

$A_{11} = 1 + 3 = 4,\qquad A_{12} = -(2 + 3) = -5,\qquad A_{13} = 2 - 1 = 1$

$A_{21} = -(-1 - 1) = 2,\qquad A_{22} = 1 - 1 = 0,\qquad A_{23} = -(1 + 1) = -2$

$A_{31} = 3 - 1 = 2,\qquad A_{32} = -(-3 - 2) = 5,\qquad A_{33} = 1 + 2 = 3$

$$adj\, A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

From (i), $X = A^{-1}B$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 - 0 + 6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore \quad \frac{1}{x} = 2, \frac{1}{y} = -1, \frac{1}{z} = 1$

Hence, $x = \frac{1}{2}, y = -1, z = 1$

Or

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$$

$|A| = 3(3 - 6) - 2(-12 - 14) + 1(12 + 7) = -9 + 52 + 19 = 62 \neq 0. \therefore A^{-1}$ exists.

Now, Cofactors of A are

$A_{11} = -3;\qquad A_{12} = 26;\qquad A_{13} = 19$

$A_{21} = 9;\qquad A_{22} = -16;\qquad A_{23} = 5$

$A_{31} = 5;\qquad A_{32} = -2;\qquad A_{33} = -11$

$$\text{adj } A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \quad \ldots(i)$$

Now, the given system of equation can be represented in matrix form,

$$A'X = B \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad X = (A')^{-1} B = (A^{-1})' B$$

$$X = \frac{1}{62} \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$X = \frac{1}{62} \begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore \quad$ **$x = 1, y = 1, z = 1$**

37. Let a, b, c be the direction ratios of the normal to the plane

Equation of the plane through pt. (1, 1, 0) is

$a(x - 1) + b(y - 1) + c(z - 0) = 0 \quad \ldots(i)$

Pt. (1, 2, 1) lies on (i) $\quad \therefore \; 0a + 1b + 1c = 0$

Pt. (–2, 2, –1) lies on (i) $\quad \therefore \; -3a + 1b - 1c = 0$

$$\frac{a}{-1-1} = \frac{-b}{0+3} = \frac{c}{0+3} = \lambda \text{ (let)}$$

$\therefore \quad a = -2\lambda, b = -3\lambda, c = 3\lambda$

Putting the values of a, b and c in (i), we have

$-2\lambda(x - 1) - 3\lambda(y - 1) + 3\lambda(z - 0) = 0$

$2(x - 1) + 3(y - 1) - 3(z) = 0$ **...[dividing both sides by $-\lambda$**

$2x - 2 + 3y - 3 - 3z = 0$

$2x + 3y - 3z - 5 = 0 \quad \ldots(ii)$

Direction ratios of the normal to the plane (i) are 2, 3, –3

Direction ratios of the given line are 3, –1, 1

Now $a_1a_2 + b_1b_2 + c_1c_2 = 6 - 3 - 3 = 0$

$\therefore$ Given line is perpendicular to the normal to the plane.

So given line is parallel to the plane (i).

The distance between the plane and given line = $\perp$ distance between Pt. (6, 3, –2) and plane (i)

$$= \left| \frac{2(6) + 3(3) - 3(-2) - 5}{\sqrt{4 + 9 + 9}} \right| \quad \ldots\left[\text{Using } \perp \text{ distance} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \right]$$

$$= \left| \frac{12 + 9 + 6 - 5}{\sqrt{22}} \right| = \frac{22}{\sqrt{22}} = \sqrt{22} \textbf{ units}$$

Or

The equations of the given lines are

$$L_1 : \vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \text{ and } L_2 : \vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$$

Here $\vec{a_1} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b_1} = \hat{i} + 2\hat{j} + 5\hat{k}$ and $\vec{a_2} = 3\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b_2} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = \hat{i}(10+10) - \hat{j}(5-15) + \hat{k}(-2-6) = 20\hat{i} + 10\hat{j} - 8\hat{k}$$

$\therefore$ Vector equation of required plane containing the given two lines is

$(\vec{r} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0$

$\Rightarrow$ $[\vec{r} - (2\hat{i} + \hat{j} - 3\hat{k})].(20\hat{i} + 10\hat{j} - 8\hat{k}) = 0$

$\Rightarrow$ $\vec{r}.(20\hat{i} + 10\hat{j} - 8\hat{k}) - (2\hat{i} + \hat{j} - 3\hat{k}).(20\hat{i} + 10\hat{j} - 8\hat{k}) = 0$

$\Rightarrow$ $\vec{r}.(20\hat{i} + 10\hat{j} - 8\hat{k}) - (40 + 10 + 24) = 0$

$\Rightarrow$ $\vec{r}.(20\hat{i} + 10\hat{j} - 8\hat{k}) = 74$

$\therefore$ $\vec{r}.(10\hat{i} + 5\hat{j} - 4\hat{k}) = 37$ **is the required Vector equation of plane.**

$(x\hat{i} + y\hat{j} + z\hat{k}).(20\hat{i} + 10\hat{j} - 8\hat{k}) = 74$

$\Rightarrow$ $\mathbf{20x + 10y - 8z = 74}$

or $\mathbf{10x + 5y - 4z = 37}$ **is the required Cartesian equation of plane.**

38. As LPP is Maximize, $Z = 200x + 20y$

Subject to the constraints: $3x + y \le 600$; $x + y \le 300$; $x - y \le 100$

$3x + y \le 600$

Let $3x + y = 600$

x	0	200
y	600	0

$x + y \le 300$

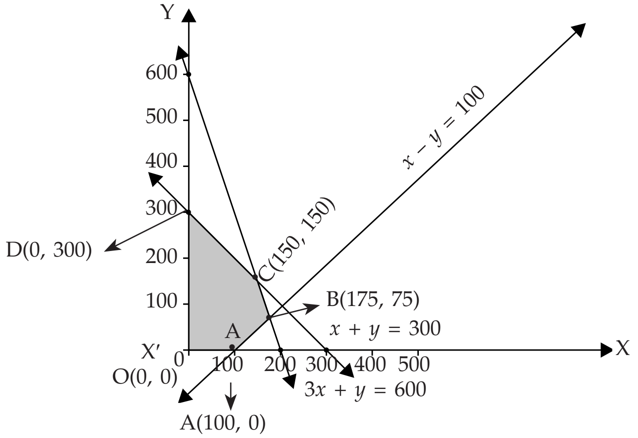
Let $x + y = 300$

x	0	300
y	300	0

$x - y \le 100$

Let $x - y = 100$

x	100	500
y	0	400



Corner Points	$Z = 200x + 20y$
A(100, 0)	₹ 20,000
B(175, 75)	₹ (35,000 + 1,500) = ₹ **36,500**
C(150, 150)	₹ (30,000 + 3,000) = ₹ 33,000
D(0, 300)	₹ 6,000
O(0, 0)	₹ 0

← **Maximum**

Maximum profit at B, *i.e.* at **x = 175 and y = 75**

and **Maximum Profit** = 200(175) + 20(75) = **₹ 36,500**

Or

Given. Minimise $Z = 2x + 3y$

Subject to the constraints:

$$2x + 3y \geq 6, \quad x - y \geq 0, \quad 2x + y \leq 8, \quad x \geq 0, \quad y \geq 0$$

Consider the following equations :

$2x + 3y = 6$

x	0	3
y	2	0

$x - y = 0$

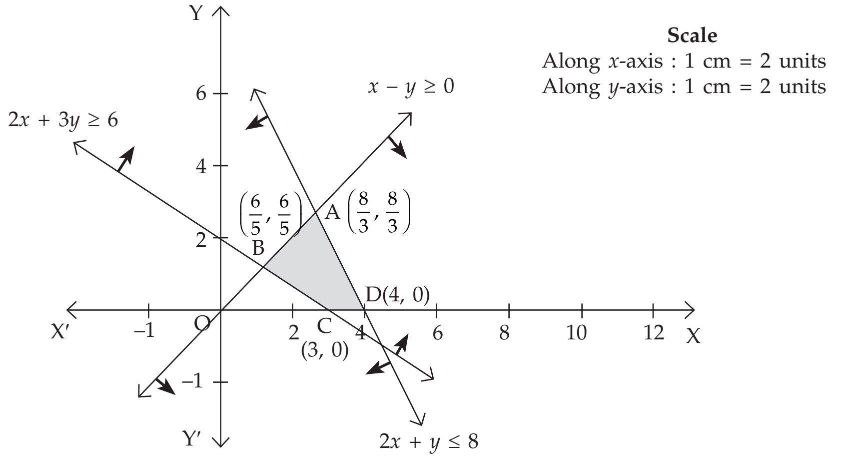
x	0	2
y	0	2

$2x + y = 8$

x	2	4
y	4	0

$x = 0, y = 0$

Feasible region of L.P.P. is bounded, as shown shaded in the graph.



Corner Points	*Value of Z = 2x + 3y*	
$A\left(\frac{8}{3}, \frac{8}{3}\right)$	$\frac{40}{3}$ (Maximum)	
$B\left(\frac{6}{5}, \frac{6}{5}\right)$	6	Minimum (Multiple Optional) Solutions
C(3, 0)	6	
D(4, 0)	8	

The feasible region is bounded and 6 is the minimum value of Z at corner. Therefore, 6 is the minimum value of Z in the feasible region at $B\left(\frac{6}{5}, \frac{6}{5}\right)$ and C(3, 0).

Hence, 6 is the minimum value of Z in the feasible region at all the points of line joining $\left(\frac{6}{5}, \frac{6}{5}\right)$ and (3, 0).

❐❐❐❐